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IKKJE PENSUM
I ACC421A

ENDÅ FLEIRE EMNE

I FUNDAMENTAL VERDIVURDERING

Føreløsing ved

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INNHALD FØRELESING 22

«TOPICS IN VALUATION»

- 1) MIDTÅRSJUSTERING
- 2) VERDIVURDERING I PYTHON

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FØRELESING 22

1.

FORMLAR

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NÅR FKE BLIR REALISERT MIDT I ÅRET

I verddivurdering er det vanleg å føresetje at kontantstraumen **blir realisert i slutten av året** slik at han skal diskonterast heile året

$$VEK_0 = (FKE_1 + VEK_1)/(1 + ekk_1)$$

→

Dersom **kontantstraumen blir realisert midt i året:**

$$VEK_0 = FKE_1 / (1 + ekk_1)^{1/2} + VEK_1 / (1 + ekk_1)$$

FKE₁ skal då diskonterast berre eit halvår

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1.1

FKE-MODELLEN

$$VEK_0 = \sum_{t=1}^T \frac{FKE_t}{(1 + ekk_1) \cdot \dots \cdot (1 + ekk_t)^{1/2}} + \frac{FKE_{T+1}}{((1 + ekk_1) \cdot \dots \cdot (1 + ekk_T)^{1/2}) \cdot (ek_{T+1} - ek_{v_{T+1}})}$$

eller

$ek_{T+1} = ek_{v_{T+1}} = \dots$

$$VEK_0 = \sum_{t=1}^T \frac{FKE_t \cdot (1 + ekk_t)^{1/2}}{(1 + ekk_1) \cdot \dots \cdot (1 + ekk_t)} + \frac{FKE_{T+1} \cdot (1 + ekk_{T+1})^{1/2}}{(1 + ekk_1) \cdot \dots \cdot (1 + ekk_T) \cdot (ek_{T+1} - ek_{v_{T+1}})}$$

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1.2

SPE-MODELLEN

$$\begin{aligned}
 VEK_0 &= EK_0 \cdot (1 + ekk_1)^{1/2} \\
 &+ \sum_{t=1}^T \frac{SPE_t \cdot (1 + ekk_t)^{1/2}}{(1 + ekk_1) \cdot \dots \cdot (1 + ekk_t)} \\
 &+ \sum_{t=1}^T \frac{(((1 + ekk_{t+1}) / (1 + ekk_t))^{1/2} - 1) \cdot EK_t \cdot (1 + ekk_t)^{1/2}}{(1 + ekk_1) \cdot \dots \cdot (1 + ekk_t)} \\
 &+ \frac{SPE_{T+1} \cdot (1 + ekk_{T+1})^{1/2}}{(1 + ekk_1) \cdot \dots \cdot (1 + ekk_T) \cdot (ek_{T+1} - ek_{T+1})}
 \end{aligned}$$

Dersom $ek_{t+1} = ek_t$, er dette leddet 0

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1.3

DØME

Framtidsrekneskap og krav:

År	0	1	2	3	4
NRE		20	11	9,6	9,792
- ΔEK		10	10	2,4	2,448
= FKE		10	1	7,2	7,344
EK	100	110	120	122,4	124,848
→ ekr		20%	10%	8%	8%
ekk		5%	5,5%	6%	6%
SPE		15	4,95	2,4	2,448
Vekst i ss					2%

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1)

FKE REALISERT I SLUTTEN AV ÅRET

1) FKE-modellen

$$\begin{aligned}
 VEK_0 &= 10/1,05 + 1/(1,05 \cdot 1,055) + 7,2/(1,05 \cdot 1,055 \cdot 1,06) \\
 &+ 7,344/(1,05 \cdot 1,055 \cdot 1,06 \cdot (0,06 - 0,02)) \\
 &= \underline{\underline{172,9}}
 \end{aligned}$$

2) SPE-modellen

$$\begin{aligned}
 VEK_0 &= 100 + 15/1,05 + 4,95/(1,05 \cdot 1,055) + 2,4/(1,05 \cdot 1,055 \cdot 1,06) \\
 &+ 2,448/(1,05 \cdot 1,055 \cdot 1,06 \cdot (0,06 - 0,02)) \\
 &= \underline{\underline{172,9}}
 \end{aligned}$$

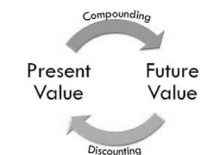
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2.1)

FKE REALISERT MIDT AV ÅRET

1) FKE-modellen

$$\begin{aligned}
 VEK_0 &= 10/1,05^{1/2} + 1/(1,05 \cdot 1,055^{1/2}) \\
 &+ 7,2/(1,05 \cdot 1,055 \cdot 1,06^{1/2}) \\
 &+ 7,344 \cdot 1,06^{1/2}/(1,05 \cdot 1,055 \cdot 1,06 \cdot (0,06 - 0,02)) \\
 &= 9,8 + 0,9 + 6,3 + 161,0 \\
 &= 9,8 + 0,9 + 167,3 \quad (\text{sjå Excel-dømet på seinare plansje}) \\
 &= \underline{\underline{178,0}}
 \end{aligned}$$



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2.2)

FKE

REALISERT MIDT AV ÅRET

2) SPE-modellen

$$\begin{aligned}
 VEK_0 &= 100 \cdot 1,05^{1/2} \\
 &+ 15/1,05^{1/2} + 4,95/(1,05 \cdot 1,055^{1/2}) + 2,4/(1,05 \cdot 1,055 \cdot 1,06^{1/2}) \\
 &+ ((1,055/1,05)^{1/2} - 1) \cdot 110/1,05^{1/2} \\
 &+ ((1,06/1,055)^{1/2} - 1) \cdot 120/(1,05 \cdot 1,055^{1/2}) \\
 &+ 2,448 \cdot 1,06^{1/2} / (1,05 \cdot 1,055 \cdot 1,06 \cdot (0,06 - 0,02)) \\
 &= 102,5 + 14,6 + 4,6 + 2,1 + 0,3 + 0,3 + 53,7 \\
 &= \underline{\underline{178,1}}
 \end{aligned}$$

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1.4

KONSTANT KRAV

$$VEK_0 = \sum_{t=1}^T \frac{FKE_t \cdot (1 + ekk_t)^{1/2}}{(1 + ekk_1) \cdot \dots \cdot (1 + ekk_t)} + \frac{FKE_{T+1} \cdot (1 + ekk_{T+1})^{1/2}}{(1 + ekk_1) \cdot \dots \cdot (1 + ekk_{T+1}) \cdot (ekv_{T+1} - ekv_{T+1})}$$

Dersom ekv_t er konstant lik ekv ,

$$VEK_0 = (1 + ekk)^{1/2} \cdot \left(\sum_{t=1}^T \frac{FKE_t}{(1 + ekk)^t} + \frac{FKE_{T+1}}{(1 + ekk)^T \cdot (ekv - ekv)} \right)$$

dvs

$$VEK_0^M = (1 + ekk)^{\frac{1}{2}} \cdot VEK_0^S$$

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MIDTÅRSJUSTERING

TILNÆRMA

1) FKE-modellen

$$\begin{aligned}
 VEK_0 &= 10/1,05 + 1/(1,05 \cdot 1,055) + 7,2/(1,05 \cdot 1,055 \cdot 1,06) \\
 &+ 7,344/(1,05 \cdot 1,055 \cdot 1,06 \cdot (0,06 - 0,02)) \\
 &= \underline{\underline{172,9}}
 \end{aligned}$$

2) Midtårsjustering

Gjennomsnittskravet er lat vi seie 5,5%

$$VEK_0 = \underline{\underline{172,9}} \cdot 1,055^{1/2} = \underline{\underline{177,6}} \approx \underline{\underline{178,0}}$$

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EXCEL

XNNV(k; FKS; dato)

1) Dersom diskonteringsrenta (k) er **konstant**, så kan vi nytte

NNV(k; FKS)

til å rekne noverdien, under føresetnad av diskontering frå slutten av perioden

Justering til midt i perioden er $NNV(k; FKS) \cdot (1 + k)^{(1/2)}$

2) Det finst også ein funksjon i Excel som brukar dei **faktiske datoane for realisering av kontantstraumen**:

XNNV(k; FKS; dato)

der datoane kan variere fritt

Engelsk:
NPV() og XNPV()

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DØME

EXCELFUNKSJONAR

DATO	31.12.2021	30.06.2022	30.06.2023	30.06.2024	30.06.2025
FKE		10	1	7,2	7,344
				183,6	
	0,0	10,0	1,0	190,8	
ekkt		5,0 %	5,5 %	6,0 %	
VEK slutten av året	172,918	9,52	0,90	162,49	
VEK midt i året	177,982	9,76	0,93	167,30	
ekkt	5,49 %				
NNV	172,918	172,918	kr 0,000	MÅL	0
NNV · (1+ekkt) ^(1/2)	177,601				
XNNV	177,615				

ekkt er det konstante gjennomsnittskravet som gjev same verdi for NNV som når ekkt varierer

Bruk av XNNV() avvik litt sidan talet på dagar avvik litt mellom datoane

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FØRELESING 22

2.

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VERDIVURDERING I PYTHON

Python is a clear and powerful object-oriented programming language



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2.1

THE BASIC

```
# Import modules which are used in the program
import matplotlib.pyplot as plt
import numpy as np

# FINANCIAL STATEMENTS t = 0
IC_0, NFD_0 = 1000, 800

# ROIC and g from 1 to T
T = 4

roic = [None, 0.20, 0.16, 0.02, 0.08]
g_ic = [None, 0.05, 0.12, 0.06, 0.03]

print()
print("THE ASSUMPTIONS:", "T =", T)
print()
print("Current financial statements:", "...IC:", IC_0, "...NFD:", NFD_0, "...E:", IC_0 - NFD_0)
print()
print("Roic:", roic)
print()
print("Growth in IC:", g_ic)
```

Imports **modules** which are used in the program

The values are given names and the other way around

For example roic is given values over time by the use of an **array**

Selected text and variables are **printed**

```
# Start with specifying the invested capital, net operating profit
# after tax, and the free cash flow from operations from 0 to T = 4
IC, NOPAT, FCFO = [IC_0], [None], [None]

for t in range(T+1):
    if t > 0:
        IC.append((1 + g_ic[t]) * IC[t-1])
        NOPAT.append(roic[t] * IC[t-1])
        FCFO.append(NOPAT[t] - (IC[t] - IC[t-1]))
    Next
print()
print("THE BUDGET:")
print()
print("IC:", IC)
print()
print("NOPAT:", NOPAT)
print()
print("FCFO:", FCFO)
print()
# The growth after T, i.e. from T+1
g = g_ic[T]
```

A **loop** fills the arrays, for example FCFO

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PYTHON PRINTS

THE ASSUMPTIONS: T = 4

Current financial statements: ...IC: 1000 ...NFD: 800 ...E: 200

Roic: [None, 0.2, 0.16, 0.02, 0.08]

Growth in IC: [None, 0.05, 0.12, 0.06, 0.03]

THE BUDGET:

IC: [1000, 1050.0, 1176.0, 1246.5600000000002, 1283.9568000000002]

NOPAT: [None, 200.0, 168.0, 23.52, 99.72480000000002]

FCFO: [None, 150.0, 42.0, -47.04000000000018, 62.32800000000003]

PYTHON PRINTS
THIS ON THE SCREEN

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CONSTANT WACC

The WACC

```
print("THE WACC:")
```

```
print()
```

```
rf, betaI, erp, crp = 0.02, 0.8, 0.05, 0.01
```

w = 0.6

```
betaD = crp/erp
```

```
kD = rf + betaD * erp
```

```
def betaE(w): return betaI + (betaI - betaD)*(1-w)/w
```

```
def kE(w): return rf + betaE(w) * erp
```

```
def wacc(w): return kE(w) * w + kD * (1 - w)
```

```
print("ASSUMPTIONS:", "rf:", rf, "erp:", erp, "crp:", crp, "betaI:", betaI)
```

```
print()
```

```
print("kE:", kE(w),"w:", w,"kD:", kD,"wacc:", wacc(w))
```

```
print()
```

THE WACC:

ASSUMPTIONS: rf: 0.02 erp: 0.05 crp: 0.01 betaI: 0.8

kE: 0.080000000000000002 w: 0.6 kD: 0.03 wacc: 0.060000000000000001

60% EQUITY WEIGHT SEEMS OKAY?

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PLOT A FIGURE

Import modules which are used in the program

```
import matplotlib.pyplot as plt
import numpy as np
```

Make a plot of wacc(w)

```
x = np.linspace(0.001, 0.999)
```

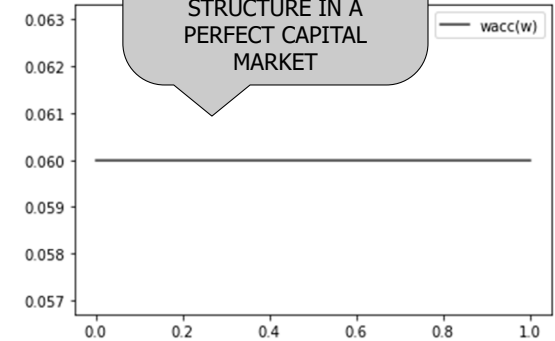
```
plt.plot(x, wacc(x), "red", label="wacc(w)")
```

```
plt.legend()
```

```
plt.show()
```

MODIGLIANI – MILLER

WACC IS INDEPENDENT OF THE CAPITAL STRUCTURE IN A PERFECT CAPITAL MARKET



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VALUATION

```
print("VALUATION")
```

The continuing value as a function of w

```
def CV(w): return FCFO[T]*(1+g)/(wacc(w) - g)
```

The enterprise value as a function of w

```
def EV(w): return FCFO[1]/(1+wacc(w)) + FCFO[2]/(1+wacc(w))**2 + FCFO[3]/(1+wacc(w))**3 + FCFO[T]/(1+wacc(w))**T + CV(w)/(1+wacc(w))**T
```

```
print()
```

```
print("The enterprise value is:", EV(w))
```

```
print()
```

```
print("The value of (net financial) debt is:", NFD_0)
```

```
print()
```

```
print("The Equity value is:", EV(w) - NFD_0)
```

```
print()
```

THE VALUATION:

The enterprise value is: 1883.7866157969324

The value of (net financial) debt is: 800

The Equity value is: 1083.7866157969324

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PLOT EV(w)

Make a plot of EV(w)

```
x = np.linspace(0.001, 0.999)
```

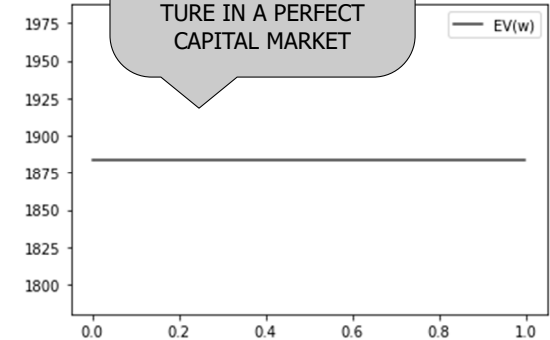
```
plt.plot(x, EV(x), "red", label="EV(w)")
```

```
plt.legend()
```

```
plt.show()
```

MODIGLIANI – MILLER

EV IS INDEPENDENT OF THE CAPITAL STRUCTURE IN A PERFECT CAPITAL MARKET



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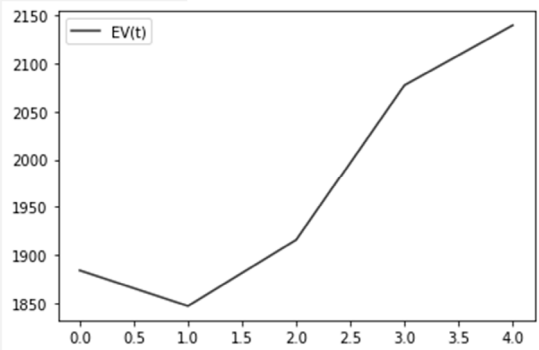
EV OVER TIME

Calculate the enterprise value over time

```

E_V = [EV(w)]
for t in range(T+1):
    if t > 0 and t < T+1:
        E_V.append(E_V[t-1] * (1 + wacc(w)) - FCF0[t])
    if t > T:
        E_V.append(E_V[t-1] * (1+g))
next
print()
print("Enterprise value over time:", E_V)
print()
# Figure: EV over time
x = range(0, T+1, 1)
plt.plot(x, E_V, "red", label="EV(t)")
plt.legend()
plt.show()
    
```

Enterprise value over time: [1883.7866157969324, 1846.8138127447485, 1915.6226415094334, 2077.5999999999995, 2139.9279999999994]



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2.2

ANALYSIS

What **should we do** to let the financing have an effect on the wacc and EV

What about uncertainty?

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1)

IMPERFECT CAPITAL MARKET

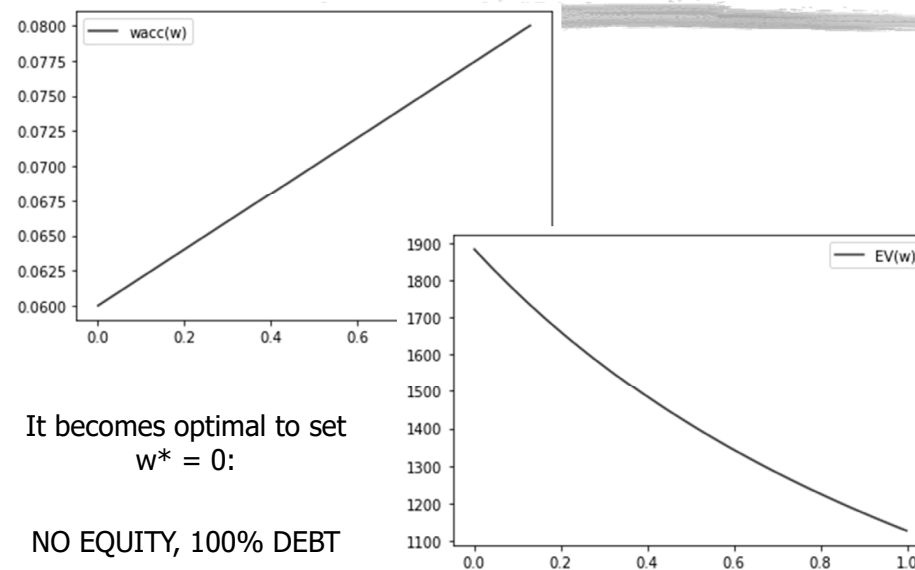
def kE(w): return rf + betaE(w) * erp + orp

CAPM holds in a perfect capital market. Adding an orp (= other risk premium) makes the cost of equity imperfect and we observe that the capital structure matter!

Assume: orp = 1%

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WACC AND EV DEPEND ON w



It becomes optimal to set $w^* = 0$:

NO EQUITY, 100% DEBT

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FIRM-SPECIFIC CRP

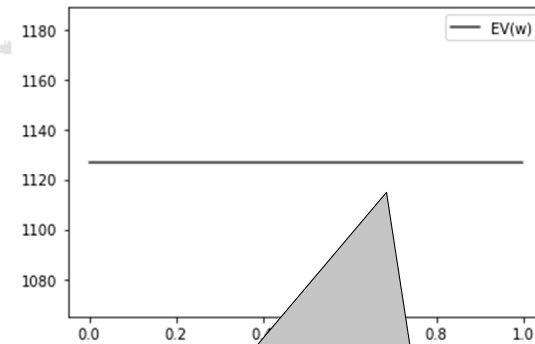
$$kD = rf + \text{betaD} * \text{erp} + (1 - \text{mrs}) * \text{crp}$$

Firm-specific crp

Assume: mrs = 0 and crp is increased to 2%

22-25

TRADEOFF



The two effects are balancing each other

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2)

SIMULATION

EXAMPLE OF ANALYSIS

-> SIMULATION

-> UNCERTAINTY OF EV

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THE CODE

```
import scipy.stats as sta
from scipy.stats.mstats import winsorize

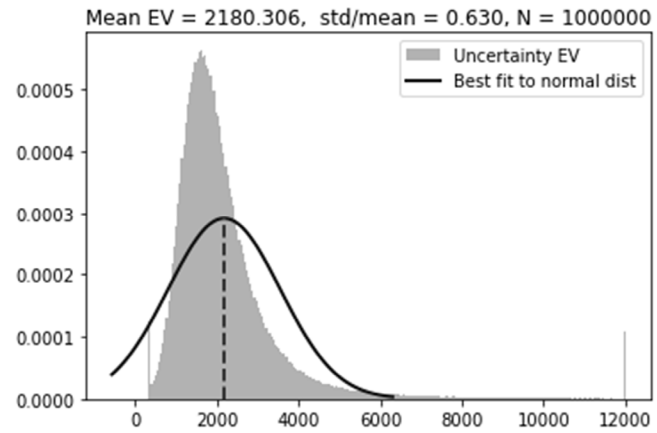
# SIMULATION
N = 1000000
mean = [rf, roic[T], g_ic[T]]
sd = [0.01, 0.02, 0.01]
rho = [0.7, 0.5, 0.6]
cov = [[sd[0]**2, rho[0]*sd[0]*sd[1], rho[1]*sd[0]*sd[2]],
        [rho[0]*sd[0]*sd[2], sd[1]**2, rho[2]*sd[1]*sd[2]],
        [rho[1]*sd[0]*sd[2], rho[2]*sd[1]*sd[2], sd[2]**2]]
data = sta.multivariate_normal.rvs(mean, cov, size=N)
rf = data[:, 0]
roic[T] = data[:, 1]
g_ic[T] = data[:, 2]
IC[T] = (1 + g_ic[T]) * IC[T-1]
NOPAT[T] = roic[T] * IC[T-1]
FCFO[T] = (NOPAT[T] - (IC[T] - IC[T-1]))
betaD = mrs*crp/erp
kD = rf + betaD * erp + (1 - mrs) * crp
betaE = betaI + (betaI - betaD)*(1-w)/w
kE = rf + betaE * erp + erp
wacc = kE * w + kD * (1 - w)
CV = FCFO[T]*(1+g)/(wacc - g_ic[T])
if any(wacc > g_ic[T]) and any(roic[T] - wacc > -0.03):
    EV = FCFO[1]/(1+wacc) + FCFO[2]/(1+wacc)**2 +
          FCFO[3]/(1+wacc)**3 + FCFO[T]/(1+wacc)**T + CV/(1+wacc)**T
else:
    EV = None
EV = list(EV)
j = len(EV) - EV.count(None)
# Fit a normal distribution to the data:
EV = winsorize(EV, (0.005, 0.005))
m_EV, sd_EV = sta.norm.fit(EV)

# Fit a normal distribution to the data:
EV = winsorize(EV, (0.005, 0.005))
m_EV, sd_EV = sta.norm.fit(EV)

# Plot the histogram.
plt.hist(EV, bins=250, density=True, alpha=0.6, color='gray', label='Uncertainty EV')
# Plot the PDF.
lowlim = m_EV-2*sd_EV
upplim = m_EV+3*sd_EV
x = np.linspace(lowlim, upplim, 100)
def p(x): return sta.norm.pdf(x, m_EV, sd_EV)
fig1 = plt.plot(x, p(x), 'blue', linewidth=2, label='Best fit to normal dist')
title = "Mean EV = %3f, std/mean = %3f, N = %0m" % (m_EV, sd_EV/m_EV, j)
plt.title(title)
plt.legend()
ymax = plt.ylim()
pmax = (0, p(m_EV))
fig2 = plt.plot([m_EV, m_EV], pmax, 'red', linestyle='dashed', linewidth=2)
plt.show()
```

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UNCERTAINTY



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2.3

CONCLUSION

Python is well suited for valuation!

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